

Workshop 1: Exploring position-time graphs moving the mouse

1. In this exercise, you'll build a simple direct manipulation model to study position-time graphs. If the Modellus program is not already open, double-click its icon.

2. Click in the **Model window**. To **create a variable** x for position, type x . (You'll notice that the variable you typed appears in green. Modellus uses different colours and font styles to display the parts of equations that you type in the Model window.)


Click on the image to see the video...



Variable x created on the Model Window

3. The next step after entering the model is to **interpret**. (The **Interpret** button signals Modellus to incorporate values you have entered in the model. Notice that in the Initial Conditions window, Modellus automatically creates entry forms for independent variables in your model.)

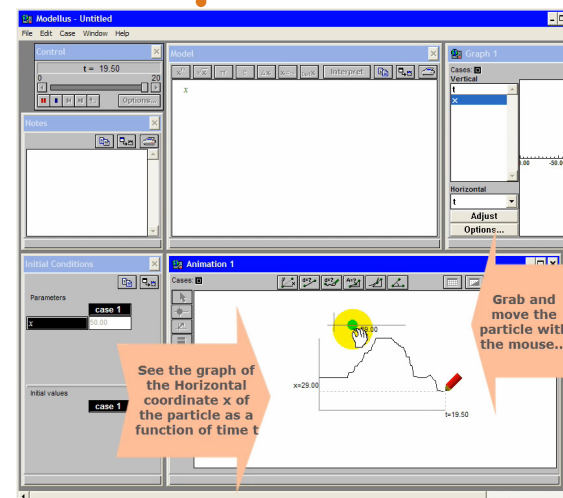
4. In the **Initial Conditions window**, assign the value "50" to the x by typing 50 in the text box.

5. Now let's represent the model as an animation. Click on the **Animation window**. From the tool bar on the left side of the window, click to select the **Particle** tool , and then left-click in the top middle of the window to place the particle there. A dialogue box appears for assigning properties to the particle.


Initial value for x

A particle with x as the horizontal coordinate

The graph of x as a function of time t ...



6. For this example, you'll put a green particle in the Animation window, and you'll **specify its Horizontal coordinate**. To specify the particle's horizontal coordinate, select x in the Horizontal list box, and then click OK.

7. From the tool bar, select the **Plotter** tool , and then left-click near the bottom of the Animation window to position the plotter there. When the properties dialogue box appears, select t (time) to specify the Horizontal coordinate and select x to specify the Vertical coordinate, and **type 0.1 in the Horizontal scale text box** (this "enlarges" the graph horizontally).

8. Click the **Run** button at the Control window. In the Animation window, the ball remains stationary while the plotter graphs x -position through time....

9. **While the simulation runs, you can alter variables to see the effects on the animation.** Grab the ball and move the mouse (press the left mouse button while you move the ball). Notice how the graph changes as you move the ball over time.

Workshop 2: Exploring position-time graphs with functions

1. In this exercise, you will see how to use **simple mathematical models** to study **position-time graphs**.

2. **Movie A** shows a particle which horizontal coordinate is described by the **function** $x = 10 \times t$. Look how the **scale** for the graph is changed, using the right button of the mouse...

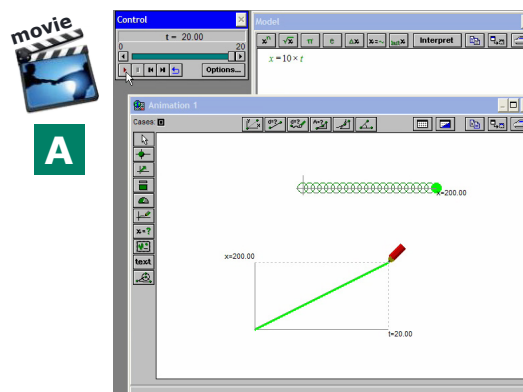
3. **Movie B** shows how to change the previous model in order to **start the motion from another position**...

4. **Movie C** shows the motion of the particle when its horizontal coordinate is described by a **quadratic function of time**... It also illustrates how to show **stroboscopic tracking**.

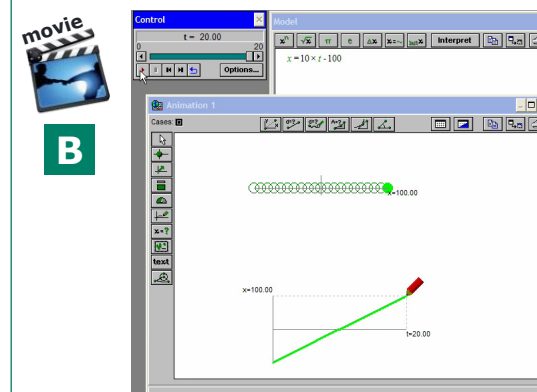
5. **Movie D** shows another **accelerating** particle, moving from **right to left**.

Use the **"Zoom In"** button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...

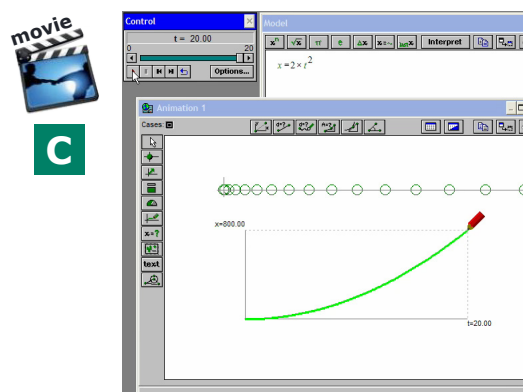
A graph of a motion with **constant speed**, starting from the origin



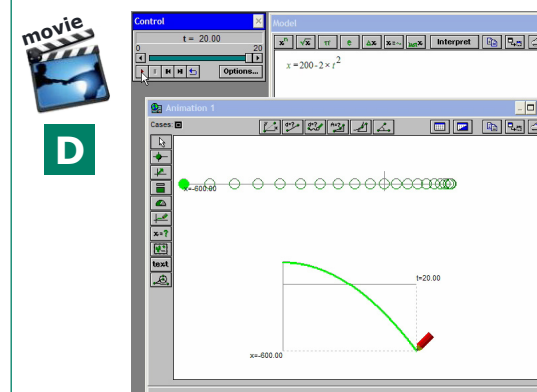
A graph of a motion with **constant speed**, starting from a negative horizontal coordinate



A graph of a motion with **increasing speed**, starting from the origin and showing **stroboscopic tracking**



A graph of a motion with **increasing speed**, starting from a positive horizontal coordinate and moving left

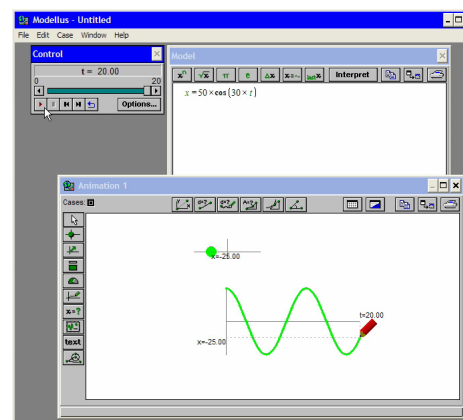


Workshop 3: Exploring horizontal oscillations

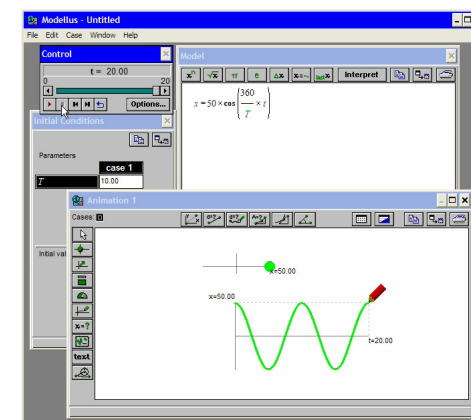
- In this exercise, you will see how to use a **co-sine function** to create an **horizontal oscillation** and the corresponding **position-time graph**.
- Movie A** shows how to create an **oscillating particle**. Look that the argument of the co-sine function is expressed in degrees (this is the default option).
- Movie B** shows how to introduce a **parameter T** (period of the oscillation) using a symbol on the model. This will be particularly useful when investigating different values for the parameter.
- Movie C** shows what must be done if we want the argument of the co-sine function to be expressed in **radians**. The Options... button on the Control panel must be used to change from degrees to radians and vice-versa. Look that $360/T$ was also change to $2\pi/T$.
- Movie D** shows how to create two set of parameters (named "Cases") for analysing two different values for the amplitude A of the oscillation. The small coloured buttons on the top-left of the Animation window can be used to change from one Case to another.

Use the "**Zoom In**" button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...

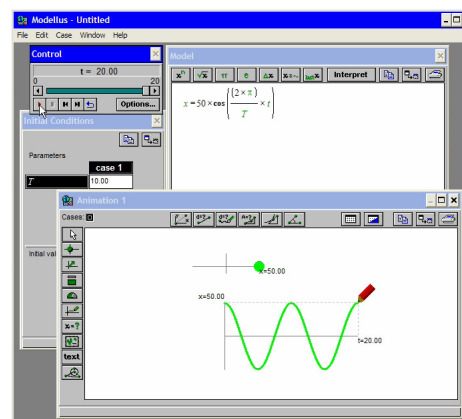
Using a co-sine function to make an horizontal oscillation



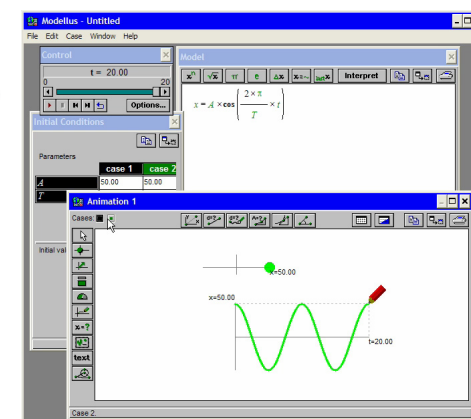
Changing the function to introduce a parameter T for the period and giving a value for T



Changing from degrees to radians, using the Options... button on Control Window



Changing the function to introduce parameter A (amplitude) and adding another "Case" to see oscillations with two different amplitudes

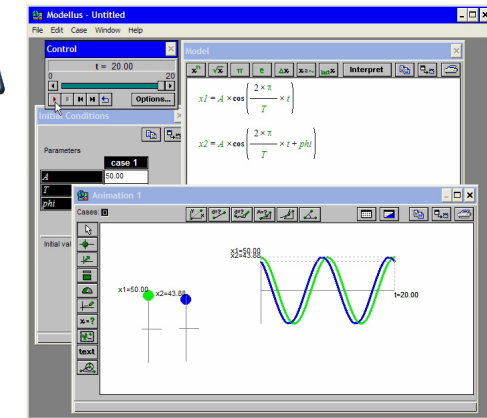
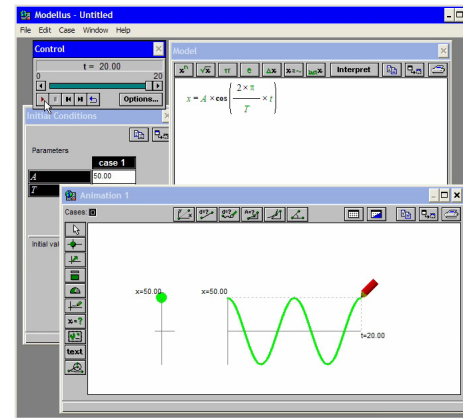


Workshop 4: Exploring vertical oscillations

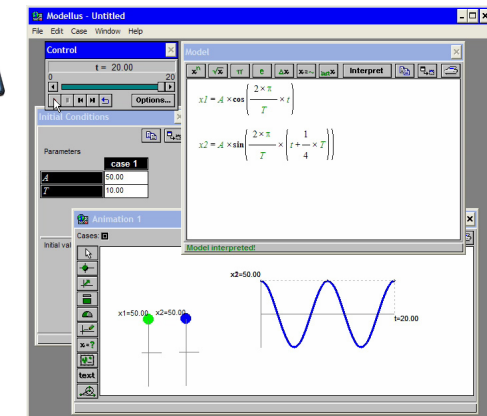
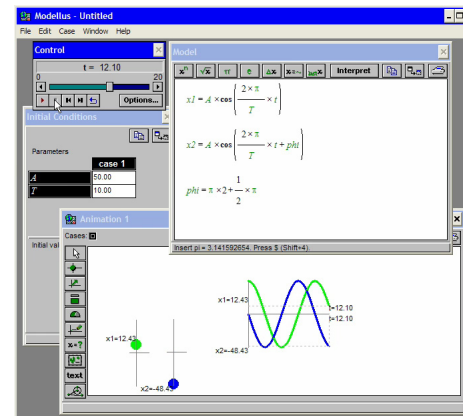
- In this exercise, you will see how to create a particle oscillating vertically and **explore different issues of this model**, particularly those related with the initial phase of the oscillation.
- Movie A** shows how to **adapt the previous model** (Workshop 3) in order to make the particle **oscillate vertically**. Usually, the symbol x is used to represent horizontal coordinates but it can be used to represent any coordinate (it could be changed to y , if you think that's relevant...).
- Movie B** shows how to create **two oscillating particles**, vertically. The vertical coordinates are represented by $x1$ and $x2$. The two particles, have different colours, as well as the graphs. One of the particles has a parameter ϕ that represent the **initial phase**. The movie illustrates how to explore different values for ϕ , introducing values on the Initial Conditions window.
- Movie C** shows a similar exploration but instead of giving values for ϕ on the Initial Conditions window the value are given directly on the Model window.
- Movie D** shows how to change one of the functions from co-sine to sine and then illustrating how the initial phase can be changed, either using an initial value for the phase or a time shift on the time function.

Use the "**Zoom In**" button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...

How to adapt a previous animation: an horizontal oscillator is changed to a vertical oscillator...



Another way of comparing oscillations with different initial phase...

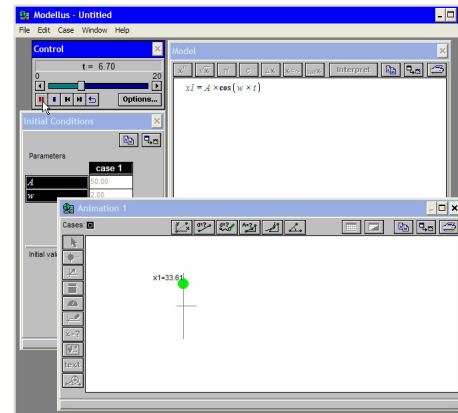


Workshop 5: Creating a transverse wave

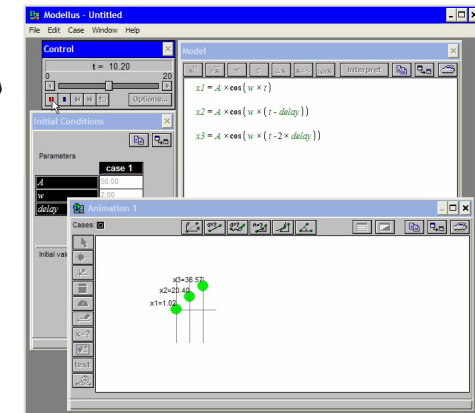
- In this exercise, you will see how to create a **set of oscillating particles** that illustrate a **transverse wave** propagating in time and space.
- Movie A** shows how to create the **first particle**. Symbols are used to define amplitude and angular frequency and the phase angle is expressed in **radians**.
- Movie B** shows how to **add two more particles**, oscillating with the same period, but **with an increasing delay**...
- Movie C** shows how to **copy and paste the oscillating function** and making a bigger delay for each particle.
- Movie D** shows how to change the model in order to introduce a **way of computing the delay as a function of the period** of the oscillation. It also illustrates the wave for different values of the delay.

Use the **"Zoom In"** button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...

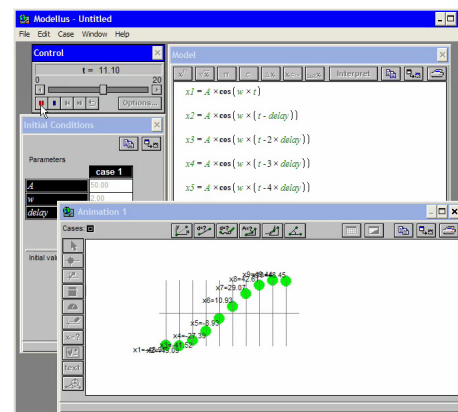
Creating a vertical oscillation, using symbols to define amplitude and angular frequency and expressing angles in radians...



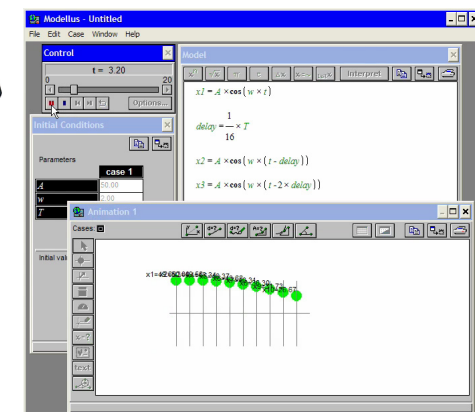
Adding two more oscillating particles, with the same amplitude and frequency, but particles 2 and 3 have a delay on the oscillation...



Copying, and pasting and the making minor ad-equated changes to have more particles, with increasing delays...



Making a change in the model in order of expressing the delay as a function of the period of the oscillation...



Workshop 6: Analysing data using an image

1. In this exercise, you will see **how to analyse a graph**, placed as an image of the background of the Animation window. The graph used was obtained with a motion sensor from a vertical oscillation on a spring.

2. **Movie A** shows **how to place an image (GIF or BMP formats) on the background of the Animation window** and how to use the first **measuring tool** (Measure coordinates) to find the **proportion between pixels and graph units** in each axis (scale factors).

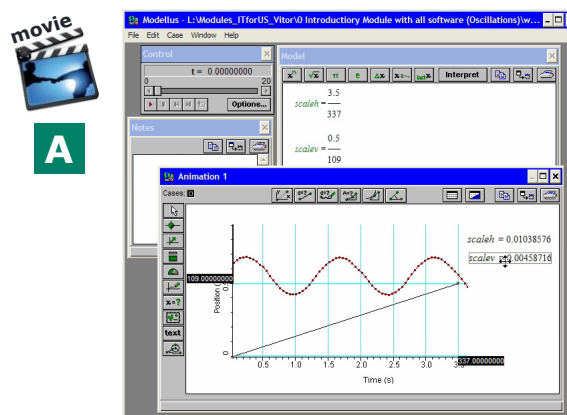
3. **Movie B** shows how to use the scales factors of the graph to **measure the period** of the oscillation. The Notes window is used to register measurements and the parameter *period* is computed on the Model window.

4. **Movie C** shows how to **measure the amplitude** of the oscillation.

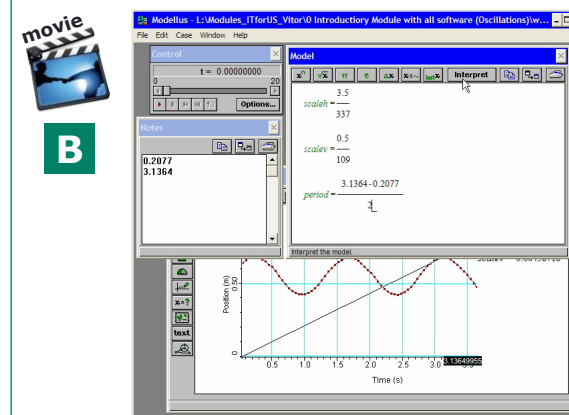
5. **Movie D** shows how to **superimpose a graph of the oscillation**, obtained from the model created with the period and the amplitude. It also illustrates how to change the model in order to introduce an **initial phase** compatible with the experimental data and how to **create an oscillating particle**, with a correct scale.

Use the **"Zoom In"** button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...

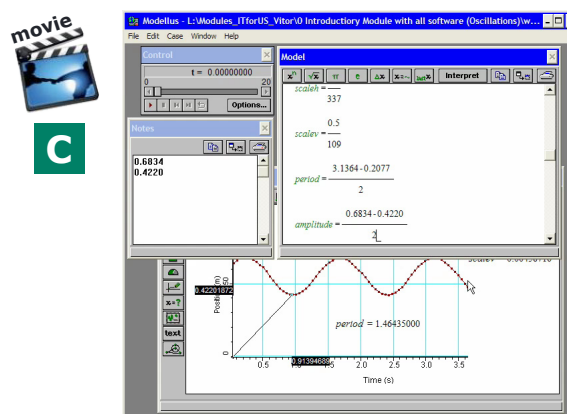
Placing an image/graph in the background of the Animation window and setting the vertical and horizontal scales of the graph



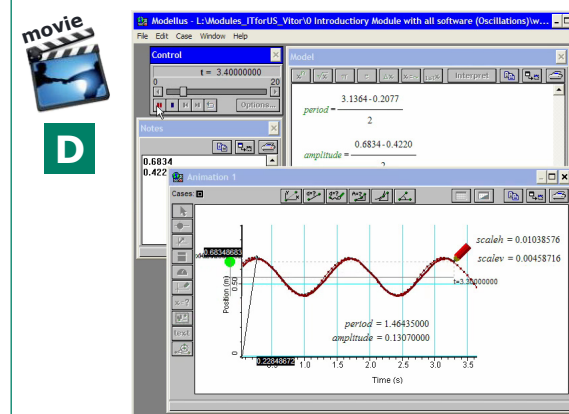
Measuring two periods and adding the period as a parameter



Measuring the amplitude of the oscillation and adding a parameter



Making the model of the oscillator... adding a graph... finding the delay... and placing a particle to represent the oscillator



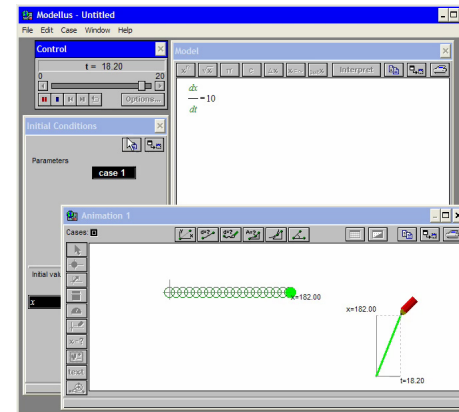
Workshop 7: Exploring inertia with differential equations

- In this exercise, you will see how to create models with **first and second order differential equations**.
- Movie A** shows a model that establish that the **instantaneous rate of change of quantity x** is equal to 10 units. Then, x is considered as an horizontal coordinate of a particle and the motion is animated on the Animation window.
- Movie B** shows how to create a **second order differential equation**: the instantaneous rate of change of quantity x is defined as quantity vx and then the instantaneous rate of change of vx is defined as ax , considered as constant. This illustrates a motion where the change of velocity is constant, i.e., an uniformly accelerated motion.
- Movie C** shows how to **create a vector to represent acceleration** and how to **manipulate** that vector in order to change the magnitude and the direction of the acceleration.
- Movie D** shows how to change the model in order to **relate acceleration to the sum of the forces** and how to create a vector to represent this sum. Usually, the scales for each animation window object must be changed in order to have objects that are visible on the available area.

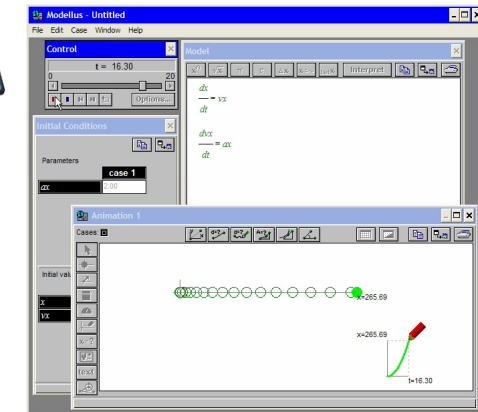
Use the "**Zoom In**" button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...



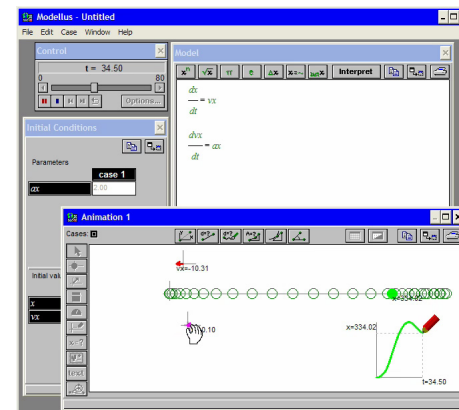
This example shows how to create a simple differential equation: the rate of change of velocity is constant and equal to 10 units...



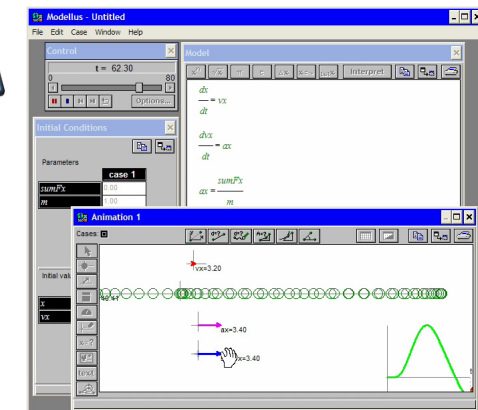
And now, the rate of change of velocity is said to be equal to the acceleration, and the acceleration is constant...



Illustrating how to create vectors to represent velocity and acceleration and how acceleration can be changed manipulating its vector...



Expressing acceleration using Newton's fundamental law of motion and creating a vector to manipulate the sum of the forces...



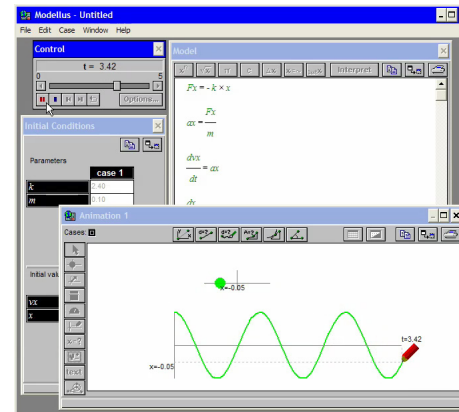
Workshop 8: Exploring oscillations with differential equations

- In this exercise, you will see how to use **differential equations to explore oscillations**, not damped and damped.
- Movie A** shows how to **define a force law** (Hooke's law), acceleration, velocity and position: the instantaneous rate of change of velocity is acceleration and the instantaneous rate of change of position is velocity.
- Movie B** shows **how to add vectors** that represent velocity and the force, as well as how to **add a graph** to show that velocity has a phase delay of 1/4 of a period.
- Movie C** shows how to **link vectors to the particle**.
- Movie D** shows how to change the model, **introducing a new term on the force law**, proportional to velocity, to illustrate how **damping** can affect the oscillation.

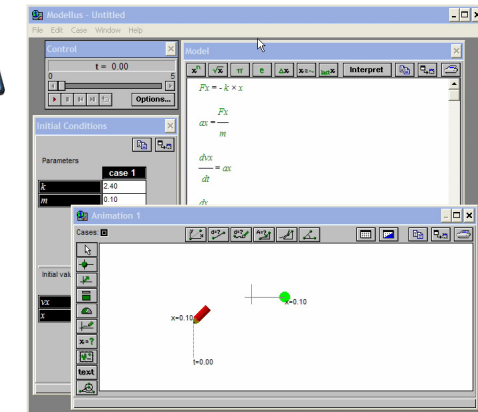
Use the **"Zoom In"** button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...



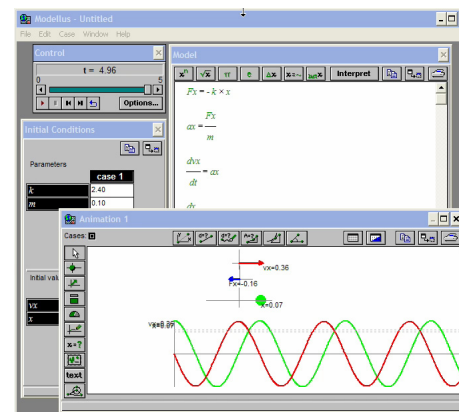
Write the force law (Hook's law), write the fundamental law of motion, write that the rate of change of velocity is acceleration, etc...



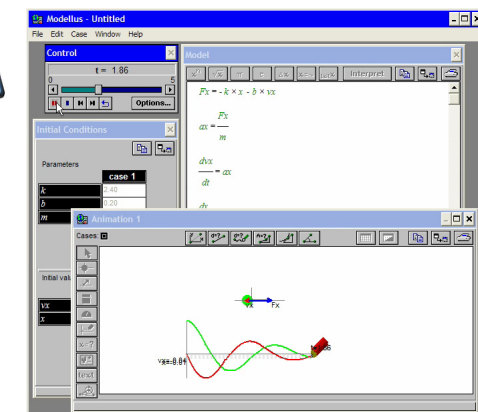
Using vectors to show force and velocity... and creating a graph with proper scales to show velocity as a function of time...



Vectors can be linked to particles (or other vectors), as shown in this example... (to release the link, edit the vector using the right button).



Changing the force law to introduce a term that depends on velocity to illustrate damping...



Workshop 9: Exploring Newton's Law of Universal Gravitation

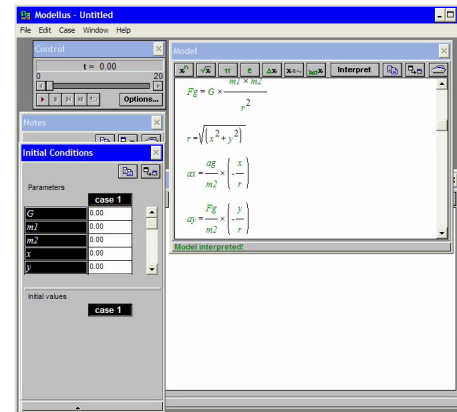
- In this exercise, you will see how to create a model of a body with a mass m_2 **orbiting around another body** of mass m_1 , assuming that body m_1 is at rest.
- Movie A** shows how to **define Newton's law of Gravitation and how to compute the acceleration components** for body m_2 from gravitational force and the coordinates of m_2 .
- Movie B** shows how to use **differential equations** to express the relation between velocity and acceleration and position and velocity, for each component. **Possible appropriate values** are given to initial values of position and velocity, as well as for parameters (universal gravitational constant and masses m_1 and m_2).
- Movie C** shows how to **change the value of parameters** in order to increase the force on the orbiting body.
- Movie D** shows how to **investigate what happens** when the initial value of the velocity is changed and how to add vectors to represent acceleration and velocity of the orbiting body.

Use the **"Zoom In"** button of Acrobat to better see each movie and the Controller  to see it frame by frame or slower...

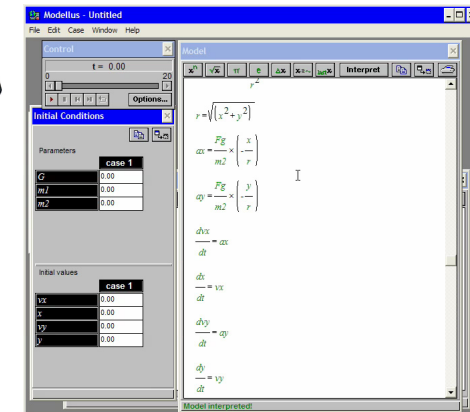
Defining the force law and expressing horizontal and vertical components of acceleration



A



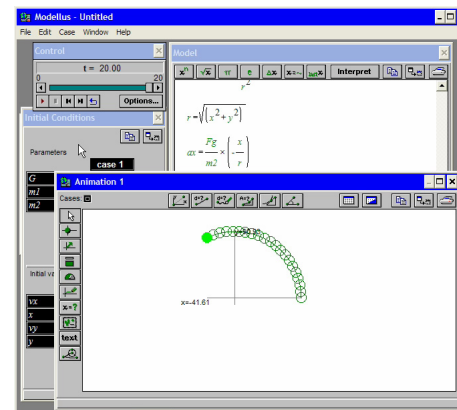
B



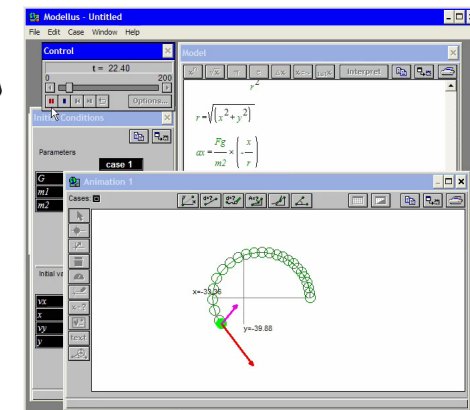
Giving appropriate arbitrary values to the universal gravitational constant, other parameters, and initial values for velocity components and position



C



D



Workshop 10: Exploring chemical kinetics and chemical equilibrium with differential equations

1. In this exercise, you will see how make a **model of a first order chemical reaction** and how to see how a chemical system reacts when the concentration of a reactant or a product is changed.

2. **Movie A** shows how to create a system of **two coupled differential equations** that represent how the concentration of the species A and B change for the reaction $A \rightarrow B$, assuming that the rate of reaction is proportional do the concentration of A. Appropriate values are given for the parameters and the initial values of A and B. The quantities A and are represented by coloured vertical bars.

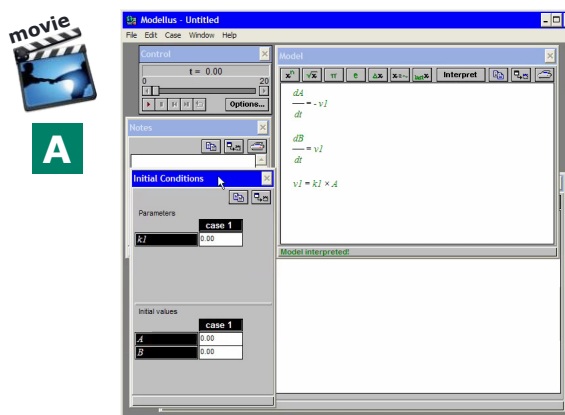
3. **Movie B** shows how to **add graphs**, with **reasonable scales**, to represent how A and B change with time.

4. **Movie C** shows how to change the equations in order to **model a reversible reaction**, $A \rightleftharpoons B$. It also shows how to run the model with different initial values.

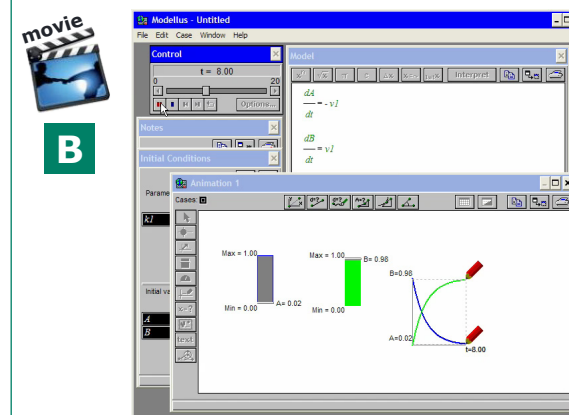
5. **Movie D** shows how to explore **how the system reacts when a sudden change** happens in one of the species of the chemical system.

Use the "**Zoom In**" button of Acrobat to better see each movie and the Controller:  to see it frame by frame or slower...

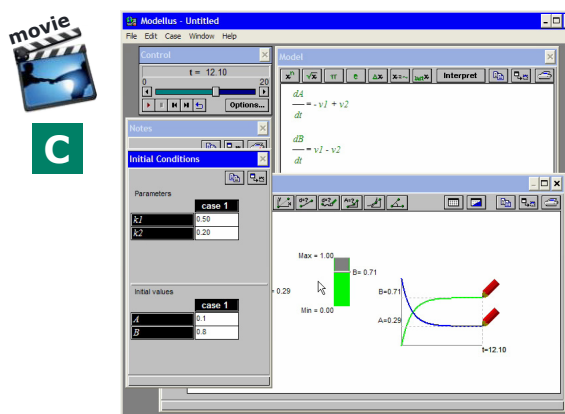
Creating a system of differential equations that describe a first order reaction and representing concentrations by coloured bars



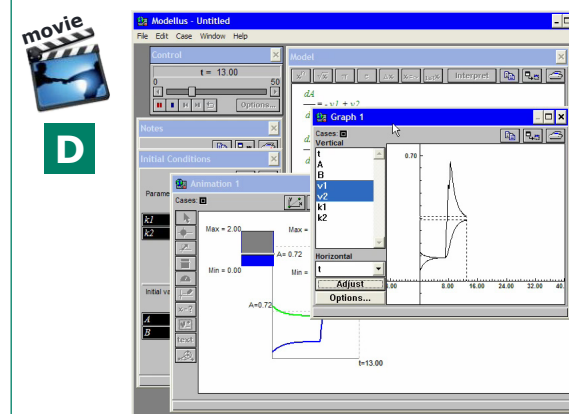
Adding two graphs to see how concentrations change with time



Changing from a non-reversible reaction to a reversible reaction... and illustration how to change initial conditions...



What happens when adding or subtracting a reactant or a product?



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